

Adding & Subtracting Directed Number using a Number line

Mathematical Form	English Sentence	Example
p	Start by drawing a number line.	Start by drawing a number line.
$+x$	$+x$ means moving x units to the right from 0.	$+6$ means moving 6 units to the right from 0.
$-x$	$-x$ means moving x units to the left from 0.	-6 means moving 6 units to the left from 0.
$+y$	$+y$ means moving y units up the y -axis from 0.	$+6$ means moving 6 units up the y -axis from 0.
$-y$	$-y$ means moving y down the y -axis from 0.	-6 means moving 6 units down the y -axis from 0.
$+(+z)$ $(+)(+)=(+)$	To add a positive number, I move z units towards the positive direction which brings us to <u>here</u> .	Given $+3+(+2)$. $+3$ means moving 3 units to the right from 0. To add a positive number, I move 2 units towards the positive direction which brings us to 5.
$+(-z)$ $(+)(-)=(-)$	To add a negative number, I move z units towards the negative direction which brings us to <u>here</u> .	Given $+3+(-2)$. $+3$ means moving 3 units to the right from 0. To add a negative number, I move 2 units towards the negative direction which brings us to 1.
$-(+z)$ $(-)(+)=(-)$	To subtract a positive number, I move z units towards the negative direction which brings us to <u>here</u> .	Given $+3-(+2)$. $+3$ means moving 3 units to the right from 0. To subtract a positive number, I move 2 units towards the negative direction which brings us to 1.
$-(-z)$ $(-)(-)=(+)$	To subtract a negative number, I move z units towards the positive direction which brings us to <u>here</u> .	Given $+3-(-2)$. $+3$ means moving 3 units to the right from 0. To subtract a negative number, I move 2 units towards the positive direction which brings us to 5.

Example 1.1

Find the values of $-7-(-4)+(-3)-(+5)$ by using a number line. Please explain your work step by step using complete sentences.

Solution

Given $-7-(-4)+(-3)-(+5)$. Start by drawing a number line. -7 means moving 7 units to the left of 0. To subtract a negative number, I move 4 units towards the positive direction which brings us to -3 . To add a negative number, I move 3 units towards the negative direction which brings us to -6 . To subtract a positive number, I move 5 units towards the negative direction which brings us to -11 .

Adding & Subtracting Positive & Negative Numbers

Mathematical Form	English Sentence	Example
p	Start with <u>the problem</u> .	Start with 4-(-3).
p->q	Because <u>of this reason</u> , I can <u>make this change</u> , which gives <u>this new equation</u> .	
(-)(-)=(+)	Because two negatives make a positive, I can replace the two negative signs with a plus sign, which gives me <u>this new equation</u> .	Start with 4-(-3). Because two negatives make a positive, I can replace the two negative signs with a plus sign, which gives me 4+3.
(+)(-)=(-)	Because adding a negative is the same as subtracting, I can replace the two different signs with a single subtraction sign, which gives me <u>this new equation</u> .	Start with 7+(-2). Because adding a negative is the same as subtracting, I can replace the two different signs with a single subtraction sign, which gives me 7-2.
(-)(+)=(-)	Because subtracting a positive is just subtracting, I can replace the two different signs with a single subtraction sign, which gives me <u>this new equation</u> .	Start with 4-(-1). Because subtracting a positive is just subtracting, I can replace the two different signs with a single subtraction sign, which gives me 4-1.
(+)(+)=(+)	Because adding a positive is just adding, I can replace the two positive signs with a single plus sign, which gives me <u>this new equation</u> .	Start with 5+(-3). Because adding a positive is just adding, I can replace the two positive signs with a single plus sign, which gives me 5+3.
+	Because ___ plus ___ equals ___, I can replace ___ with ___, which gives me <u>this new equation</u> .	Start with 7+3-2. Because 7 plus 3 equals 10, I can replace 7+3 with 10, which gives me 10-2.
-	Because ___ minus ___ equals ___, I can replace ___ with ___, which gives me <u>this new equation</u> .	Start with 10-2. Because 10 minus 2 equals 8, I can replace 10-2 with 8, which gives me 8.
-x-y	Because both numbers are negative, I add the numbers but keep the sign. Because <u>x</u> plus <u>y</u> equals <u>z</u> , I can replace <u>-x-y</u> with <u>-z</u> , which gives me <u>this new equation</u> .	Start with -7-8+3. Because both numbers are negative, I add the numbers but keep the sign. Because 7 plus 8 equals 15, I can replace -7-8 with -15, which gives me -15+3.
-x+y	Because addition is commutative, I can switch the order, which gives me <u>this new equation</u> .	Start with -15+3. Because addition is commutative, I can switch the order, which gives me 3-15.
small-big	When subtracting a larger number from a smaller one, the result will be negative. Because <u>big</u> minus <u>small</u> equals <u>answer</u> , I can replace <u>small-big</u> with <u>-answer</u> , which gives <u>this new equation</u> .	Start with 3-15. When subtracting a larger number from a smaller one, the result will be negative. Because 15 minus 3 equals 12, I can replace 3-15 with -12, which gives -12.
q	My final answer is <u>the answer</u> .	My final answer is -12.

Example 1.2

Simplify and evaluate $-7-(-4)+(-3)-(+5)$. Please explain your work step by step using complete sentences.

Solution

Start with $-7-(-4)+(-3)-(+5)$. Because two negatives make a positive, I can replace the two negative signs with a plus sign, which gives me $-7+4+(-3)-(+5)$. Because adding a negative is the same as subtracting, I can replace the two different signs with a single subtraction sign, which gives me $-7+4-3-(+5)$. Because subtracting a positive is just subtracting, I can replace the two different signs with a single subtraction sign, which gives me $-7+4-3-5$. Because both 3 and 5 are negative, I add the numbers but keep the sign. Because 3 plus 5 equals 8, I can replace $-3-5$ with -8 , which gives me $-7+4-8$. Because addition is commutative, I can switch the order, which gives me $4-7-8$. Because 7 plus 8 equals 15, I can replace $-7-8$ with -15 , which gives me $4-15$. When subtracting a larger number from a smaller one, the result will be negative. Because 15 minus 4 equals 11, I can replace $4-15$ with -11 , which gives -11 . My final answer is -11 .

Mixed Addition and Subtraction of Directed Numbers with Brackets

Mathematical Symbols	English Sentence
()	Evaluate the values in the bracket and then remove the bracket.
[]	Evaluate the values in the square bracket, and then remove the bracket.
{ }	Evaluate the values in the square bracket, and then remove the bracket.
{[()]}	Evaluate the values of the expression by removing the brackets inner out. First, the round bracket, then the square bracket. Finally the curly bracket.

Example 1.3

Evaluate $3-(5-3)$. Please explain your work step by step using complete sentences.

Solution

Evaluate the value of $(5-3)$, and then remove the bracket. Therefore, it becomes $3-2$, which is equals to 1.

Example 1.4

Evaluate $3-(3-5)$. Please explain your work step by step using complete sentences.

Solution

Evaluate the values within the brackets. Therefore the expression would become $3-(-2)$. Since subtracting a negative number would become plus. So the expression becomes $3+2$, which equals to 5.

Example 1.5

Evaluate $-10-[3+(-2-1)]$. Please explain your work step by step using complete sentences.

Solution

Start with $-10-[3+(-2-1)]$. Evaluate the values by removing the bracket inner out. First, the round bracket $(-2-1)$, as $-2 - 1$ is the same as -3 , the expression becomes $-10-[3+ (-3)]$. A negative times a positive is negative, so $+(-3)$ is the same as -3 . The expression now becomes $-10-[3-3]$. While $3-3$ is a 0, the expression is the same as $-10-[0]$. Therefore, it is -10 .

Alternative: Evaluate the values by removing the bracket inner out. First, the round bracket $(-2-1)$, as $-2 - 1$ is the same as -3 , the expression becomes $-10-[3+ (-3)]$. Since adding a negative number become minus, so $+(-3)$ gives -3 . The expression now becomes $-10-[3-3]$. While $3-3$ is a 0, so we have $-10-[0]$, and the answer is -10 .

Example 1.6

Evaluate $-\left[\left(1 + \frac{3}{4}\right) - \frac{5}{6}\right]$. Please explain your work step by step using complete sentences.

Solution

Start with $-\left[\left(1 + \frac{3}{4}\right) - \frac{5}{6}\right]$. Evaluate the values by removing the bracket inner out. First the round bracket $\left(1 + \frac{3}{4}\right)$. To add a 1 and $\frac{3}{4}$, the numbers must have a common denominator. 1 is changed (transformed) into $\frac{4}{4}$. The expressions $1 + \frac{3}{4}$ then becomes $\frac{4}{4} + \frac{3}{4}$, which is $\frac{7}{4}$ in total. The expression $-\left[\left(1 + \frac{3}{4}\right) - \frac{5}{6}\right]$ becomes $-\left[\frac{7}{4} - \frac{5}{6}\right]$. Next, to evaluate $\frac{7}{4} - \frac{5}{6}$, the fractions must be of the same denominator. As 12 is the least common multiple of 4 and 6, we multiply $\frac{7}{4}$ to $\frac{21}{12}$ and $\frac{5}{6}$ to $\frac{10}{12}$. The expression $-\left[\frac{7}{4} - \frac{5}{6}\right]$ becomes $-\left[\frac{21}{12} - \frac{10}{12}\right]$. While $\frac{21}{12} - \frac{10}{12}$ is the same as $\frac{11}{12}$. The answer is $-\frac{11}{12}$.

Multiplying & Dividing Positive & Negative Numbers

Mathematical Symbols	English Sentence	Example
$(-)(-)=(+)$	Two negatives make a positive, and ____ times ____ equals ____, so now I have ____.	Start with $(-2)(-5)$. Two negatives make a positive, and 2 times 5 equals 10, so now I have 10.
$(+)(-)=(-)$ $(-)(+)=(-)$	A negative times a positive is negative, and ____ times ____ equals ____, so now I have ____.	Start with $(+4)(-7)$. A negative times a positive is negative, and 4 times 7 is 28, so now I have -28.
$(+)(+)=(+)$	Two positives make a positive, and ____ times ____ equals ____, so now I have ____.	Start with $(+2)(+5)$. Two positives make a positive, and 2 times 5 equals 10, so now I have 10.
$(-) \div (-) = (+)$	Two negatives make a positive, and ____ divided by ____ equals ____, so now I have ____.	Start with $(-2) \div (-4)$. Two negatives make a positive, and 2 divided by 4 equals .5, so now I have .5.
$(+) \div (-) = (-)$ $(-) \div (+) = (-)$	A negative and a positive make a negative, and ____ divided by ____ equals ____, so now I have ____.	Start with $\frac{-4}{16}$. A negative and a positive is negative, and 4 divided by 16 is .25, so now I have -.25.
$(+) \div (+) = (+)$	Two positives make a positive, and ____ divided by ____ equals ____, so now I have ____.	Start with $(+8) \div (+2)$. Two positives make a positive, and 8 times 2 equals 4, so now I have 4.

Example 1.7

Evaluate $(-3)(-4)$. Please explain your work step by step using complete sentences.

Solution

Start with $(-3)(-4)$. Two negatives make a positive, and 3 times 4 equals 12, so now I have 12.